OBJECTIVE MATHEMATICS

Volume 2

Descriptive Test Series

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CHAPTER-4 : APPLICATION OF DERIVATIVES

UNIT TEST-1

1. Let a curve $y = f(x)$, $x \in (0, \infty)$ pass through the points $P\left(1, \frac{3}{2}\right)$ and $Q\left(a, \frac{1}{2}\right)$. If the tangent at any point $R(b, f(b))$ to the given curve cuts the y-axis at the point $S(0, c)$ such that $bc = 3$, then $(PQ)^2$ is equal to ______.

2. Let M and N be the number of points on the curve $y^5 - 9xy + 2x = 0$, where the tangents to the curve are parallel to x-axis and y-axis, respectively. Then the value of $M + N$ equals ______.

3. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point $(2, -1)$, then $|2a + 9b|$ is equal to _____ .

Hints and Solutions

1. (5)

$Y - y = m(X - x)$, $m = \frac{dy}{dx}$

Put $X = 0$

$Y = y - mx \Rightarrow x(y - mx) = 3$

or $y - \frac{ydx}{dx} = \frac{3}{x}$

or $\frac{ydx - Xdy}{X^2} = \frac{3dx}{x} \cdot \frac{1}{x^2}$

or $d\left(\frac{-y}{x}\right) = 3d\left(\frac{x^{-2}}{-2}\right)$

$\Rightarrow \frac{y}{x} = \frac{3}{2x^2} + C$

$C = 0$

$\frac{1}{2\alpha} = \frac{3}{2\alpha^2}$

or $\alpha = 3$

$P\left(1, \frac{3}{2}\right) Q\left(3, \frac{1}{2}\right)$

$(PQ)^2 = 4 + 1 = 5$

2. (2)

Here equation of curve is

$y^5 - 9xy + 2x = 0$ ... (i)

On differentiating: $5y^4 \frac{dy}{dx} - 9y - 9x\frac{dy}{dx} + 2 = 0$

$\therefore \frac{dy}{dx} - = \frac{9y - 2}{5y^4 - 9x}$

When tangents are parallel to x-axis then $9y - 2 = 0$

$\therefore M = 1.$

For tangent perpendicular to x-axis

$5y^4 - 9x = 0$ ...(ii)

From equation (i) and equation (ii) we get only one point.

$\therefore N = 1.$

$\therefore M + N = 2.$

3. (195)

Slope of tangent to curve $y = 5x^2 + 2x - 25$

$= m = \left(\frac{dy}{dx}\right)_{at(2,-1)} = 22$

$\therefore$ Equation of tangent : $y + 1 = 22(x - 2)$

$\therefore y = 22x - 45.$

Slope of tangent to $y = x^3 - x^2 + x$ at point (a, b)

$= 3a^2 - 2a + 1$

$3a^2 - 2a + 1 = 22$

$3a2 - 2a - 21 = 0$

$\therefore\ a = 3 \text{ or } -\frac{7}{3}$

Also $b = a^3 - a^2 + a$

Then $(a, b) = (3, 21)$ or $\left(-\frac{7}{3}, -\frac{151}{9}\right)$

$\left(-\frac{7}{3}, -\frac{151}{9}\right)$ does not satisfy the equation of tangent

$\therefore\ a = 3, b = 21 \quad \therefore \quad |2a + 9b| = 195$